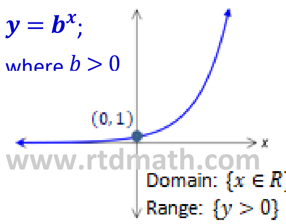
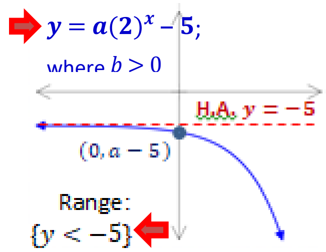
1 First consider graph of $f(x)$, since the domain of the inverse is the range of $f(x)$. (They switch!) So let's re-phrase this question … what's the RANGE of $f(x)$?

$y = b^x$; where $b > 0$



$y = a(2)^x - 5$; where $b > 0$



So the domain of the inverse is $x < -5$ ANSWER: **C**

2 $(b+1)^{\frac{1}{2}} = 3m$ First step – *convert to exp. form*

$\left((b+1)^{\frac{1}{2}}\right)^2 = (3m)^2$ To isolate b, square both sides (get that exp 1/'2 to be "1")

$\left((b+1)^{\frac{1}{2}}\right)^{2} = 9m^2$

$b + 1 = 9m^2$

$-1 \quad -1$

$b = 9m^2 - 1$ ANSWER: **A**

NR#1 $f(0) = a(b^0) + d$ For y-intercept, set $x = 0$

$= a(1) + d$

$= a + d$ ← CODE 4

$y = 0$ ← Asymptote of basic exp. graph $y = b^x$, shown above in answer to #1

$y = d$ ← Horiz. Asymptote after being shifted vertically "d" units.

CODES 7 and 2 (either order) ANSWER: **472** or **427**

3 $ax + b > 0$ $f(x) = log_b(\underbrace{ax+b})$

Isolate x Whatever we're "logging" must be greater than 0. Can't log 0, can't log negatives!

$ax > -b$

$x > \frac{-b}{a}$ ANSWER: **A**

4 $f(0) = log_b(a(0) + b)$ For y-intercept, set $x = 0$

$= log_b(0 + b)$

$= log_b(b)$ ← $b^{\square} = b$?

$= 1$ ANSWER: **C**

5 $\frac{3^{x^2+x}}{(3^3)^{3x-1}} = 3(3^{-2})^{x-2}$ *Re-write all three terms in base 3*

$\frac{3^{x^2+x}}{3^{9x-3}} = 3^1 * 3^{-2x+4}$ *Simplify using exponent rules*

$3^{x^2+x-(9x-3)} = 3^{-2x+4}$ *Once both sides are fully simplified – set the exponents equal*

$x^2 + x - 9x + 3 = -2x + 4$

$x^2 - 6x \boxed{-2} = 0$ ANSWER: **C**

c value!

6 $a^b = \frac{c}{2}$ ← First – isolate power term by dividing both sides by 2

$log_a\left(\frac{c}{2}\right) = b$ Then convert to log form to isolate "b"

$b = log_a\left(\frac{c}{2}\right)$ ANSWER: **D**

NR#2

Check **Statement 1:**

$f(0) = a(2^{(0)-1}) + d$ ← *For y-int, set* $x = 0$

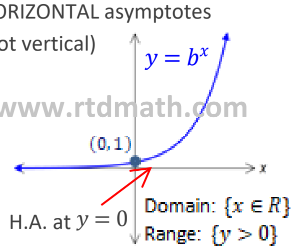
$= a(2^{-1}) + d$

$= a\left(\frac{1}{2}\right) + d$

$= \frac{a}{2} + d$ **Statement 1 is TRUE**

Check **Statement 2:**

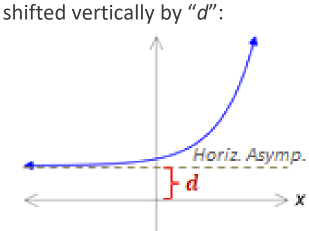
Exponential functions have HORIZONTAL asymptotes (not vertical)



Statement 2 is FALSE

Check **Statement 3:**

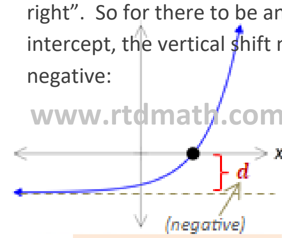
Consider the graph on the left shifted vertically by "*d*":



Statement 3 is TRUE

Check **Statement 4:**

Since $a > 0$ the graph must "rise right". So for there to be an x-intercept, the vertical shift must be negative:



Statement 3 is FALSE

There ***can be*** *an* x*-intercept*

Check **Statement 5:**

See rationale for statement 4

Statement 5 is TRUE

Check **Statement 6:**

The domain of the inverse is the RANGE of $y = f(x)$ ← *Which is NOT* $y \in R$

Statement 6 is FALSE

ANSWER: **135**

7

$y = ab^{\frac{t}{p}}$ ← *Use this formula, with $p = 1$ since we want a growth rate per 1 year*

$1239220 = 403\,319(b)^{45}$

end amount; Initial amount; # of years from 1971 to 2016

$\frac{1239220}{403\,319} = b^{45}$ *Isolate the power term*

$b^{45} \approx 3.072555$ *Take the "45th root" of both sides to solve for b*

$\sqrt[45]{b^{45}} \approx \sqrt[45]{3.072555}$

$b \approx \underbrace{1.025}_{\text{growth rate}}$ $b = 1 +$ *growth rate*

growth rate $= 2.5\%$

ANSWER: **B**

8

$y = ab^{\frac{t}{p}}$ ← *Use this formula, find p*

$1160 = 3600\left(\frac{1}{2}\right)^{\frac{24}{p}}$

← We were given amount after 1 day (answers are all in hours)

end amount; Initial amount; Half-life problem; **FIND p**

$\frac{1160}{3600} = \left(\frac{1}{2}\right)^{\frac{24}{p}}$ *Isolate the power term*

$log_{1/2}\left(\frac{1160}{3600}\right) = \frac{24}{p}$ *Convert to log form (or, if more to your liking, "log both sides")*

$p = \frac{24}{log_{1/2}\left(\frac{1160}{3600}\right)}$ ➡ $p \approx 14.7$ (**hours**, *since t was in hrs.*)

ANSWER: **B**

9

$y = ab^{\frac{t}{p}}$ ← *Use this formula, where $p = 1$*

$1127.84 = 1000(b)^2$ ← After two years

end amount; Initial amount; **FIND b**

$\frac{1127.84}{1000} = b^2$ *Isolate the power term*

$b^2 = 1.12784$

$b = \sqrt{1.12784}$ *Square root both sides*

$b \approx 1.062$ *Use this in equation*

Now, use $b = 1.062$ to solve for "t" when $y = 2000$.

$y = 1127.84(1.062)^t$

$2000 = 1127.84(1.062)^t$

$\frac{2000}{1127.84} = (1.062)^t$

Convert to log form to solve for t:

$t = log_{1.062}\left(\frac{2000}{1127.84}\right)$

$t \approx 9.5$ ANSWER: **D**

NR #3

Convert each to exp. form:

$a^{2b-1} = 8$ $\quad 2^b = a$

Substitute "2^b" for a in first equation

$(2^b)^{2b-1} = 8$

Re-write using base 2:

$2^{2b^2-b} = 2^3$

Set exponents equal and solve:

$2b^2 - b = 3$ ➡ $2b^2 - b - 3 = 0$

$(2b-3)(b+1) = 0$ $\quad b = 3/2$ or $\cancel{-1}$ *extraneous*

ANSWER: **1.5**

10.

Get the log terms on the same side:

$log_5 a - 2log_5 c = b$

Apply log laws on left side:

$log_5 a - log_5 c^2 = b$

$log_5 \frac{a}{c^2} = b$

Convert to exp. form to isolate "a":

$5^b = \frac{a}{c^2}$ ➡ $a = 5^b c^2$ ANSWER: **B**

NR #4

Convert to exp. form to solve:

$(x+1)^2 = 2x + 10$

Expand left side, move right side terms over

$x^2 + 2x + 1 - 2x - 10 = 0$

Simplify and solve resulting quadratic:

$x^2 - 9 = 0$ ➡ $x^2 = 9$ ANSWER: **13**

$x = 3$ or $\cancel{-3}$ ***Extraneous*** *– subbing into the original equation would lead to logging a negative (illegal)*

11.

Convert each to exp. form:
(first isolate power term on 2nd one)

$8^{2/3} = m$ $\quad 2^n = 5/3$ ➡ $n = log_2\left(\frac{5}{3}\right)$

$m = 4$

Now, substitute into: $log_m n^m$

$= 4log_4\left(log_2\left(\frac{5}{3}\right)\right)$ *Put into your calc*

```
4log₄(log₂(5/3))
          -0.8806616548
```

≈ -0.88 ANSWER: **C**

12.

Split up 360 into multiples of **5** and **2** *(sinc... we're given those logs)*

$= log_3(5 * 8 * 9)$

Split up using log laws:

$= log_3 5 + log_3 8 + log_3 9$

$3^{\square} = 2$?

$= log_3 5 + log_3 2^3 + 2$

$= log_3 5 + 3log_3 2 + 2$

$= \boldsymbol{a + 3b + 2}$ ANSWER: **D**

NR #5

Convert to exp. form (using base 2, as indicated) to solve:

$log_2(5^m) = 3m - 1$

Use log laws on left side:

$mlog_2 5 = 3m - 1$

Re-arrange get "m" terms on same side:

$1 = 3m - mlog_2 5$ *Factor out "m" to isolate*

$1 = m(3 - log_2 5)$

$\frac{1}{(3 - log_2 5)} = m$ ANSWER: **35**

13.

Step 1 #'s in front become exponents:

$log_2(4m)^{\frac{1}{2}} = log_2 n + log_2 p^2$

Step 2 Combine LS to single log:

$log_2\sqrt{4m} = log_2(np^2)$

Step 3 Since same base – drop logs:

$\sqrt{4m} = np^2$

➡ $2\sqrt{m} = np^2$ ➡ $\sqrt{m} = \frac{np^2}{2}$

Step 4 Square both sides to isolate m

$(\sqrt{m})^2 = \left(\frac{np^2}{2}\right)^2$ ➡ $m = \frac{\boldsymbol{n^2p^4}}{\boldsymbol{4}}$ ANSWER: **A**

NR #6 Log laws / combine to single log:

$\log_5\left(\frac{x}{x-1}\right) = 3$

Convert to exp. form:

$5^3 = \frac{x}{x-1}$

Cross-multiply to solve:

$125(x-1) = x$

$125x - 125 = x$

$125x - x - 125 = 0$

$\mathbf{124}x - 125 = 0$

ANSWER: **124**

14. Log laws / #'s *in front* become exponents:

$= \log(4A)^2 - \log 2^3 + \log(A^{1/2})^6$

(Also distribute – sign through brackets)

$= \log(4^2A^2) - \log 8 + \log A^3$

Log laws / combine to single log:

$\log \frac{\quad}{\quad}$ *Pos (+) logs go up top* / *Neg (−) logs down here*

$= \log\left(\frac{16A^2 * A^3}{8}\right)$

Simplify inside brackets:

$= \boldsymbol{\log(2A^5)}$ ANSWER: **B**

NR #7 Express Richter values as exponents of 10:

$\frac{10^{higher\ Richter}}{10^{lower}} =$ *"times as intense"*

$\frac{10^{7.3}}{10^x} = 4$

Van. Island Richter is higher / goes on top

If Q. Charlotte had 1/4th intensity, then Van. Island was 4 times as intense

Unknown / Richter for Q.Charlotte earthquake was smaller (goes on bottom)

$10^{7.3-x} = 4$

Convert to log form to solve:

$\log_{10}4 = 7.3 - x$ ➜ $x = 7.3 - \log 10$

➜ $\boldsymbol{x \approx 6.7}$ ANSWER: **6.7**

15. Convert first log to exp. form:

$m^5 = n$ *We can now substitute into second log*

$\log_m(n^{\frac{3}{4}} m^2)$

Becomes….

$\log_m((m^5)^{\frac{3}{4}} m^2)$

Now simplify:

Multiply exponents: $5 * \frac{3}{4}$

$= \log_m(m^{\frac{15}{4}} m^2)$

Add exponents: $\frac{15}{4} + 2 = \frac{15}{4} + \frac{8}{4}$

$= \log_m(m^{\frac{23}{4}})$

$= \boldsymbol{\frac{23}{4}}$ ANSWER: **D**

16. Combine L.S. to single log:

$\log_3[(x-3)(x-2)] = 2$

Then convert to exp. form:

$3^2 = (x-3)(x-2)$

Solve resulting quadratic equation.

But watch for extraneous solutions!

$9 = x^2 - 5x + 6$

$0 = x^2 - 5x - \mathbf{3}$

ANSWER: **A**

Written #1

First bullet The horiz. asymptote represents the vert. shift, "k"

So…. $\boldsymbol{k = 2}$

Sub into equation and solve for "a"

$y = a(3)^x + 2$

Use any given point to solve for "a"

$7 = a(3)^0 + 2$ ➜ $7 = a(1) + 2$ ➜ $\boldsymbol{a = 5}$

So equation is:

$\boldsymbol{y = 5(3)^x + 2}$

Second For inverse, switch x and y

$f(x)$: $y = 3(2)^x - 1$

$g(x)$: $x = 3(2)^y - 1$

Then solve for y: Isolate power term, 2^y, to convert to log form

$x + 1 = 3(2)^y$ ➜ $\frac{x+1}{3} = 2^y$ ➜ $\log_2\left(\frac{x+1}{3}\right) = y$ ➜

$\boldsymbol{y = \log_2\left(\frac{x+1}{3}\right)}$ or $\boldsymbol{y = \log_2\left[\frac{1}{3}(x+1)\right]}$

Third bullet $g(x)$ has an x-int when $y = 0$

Using $x = 3(2)^y - 1$ *equation:*

$x = 3(2)^0 - 1$ x-intercept

$x = 3(1) - 1$ ➜ $\boldsymbol{x = 2}$

Using $y = \log_2\left(\frac{x+1}{3}\right)$ *equation:*

$0 = \log_2\left(\frac{x+1}{3}\right)$

➜ $2^0 = \frac{x+1}{3}$

Same result!

$3 = x + 1$

$g(x)$ has a y-int when $x = 0$

Using $y = \log_2\left(\frac{x+1}{3}\right)$ *equation:*

$y = \log_2\left(\frac{0+1}{3}\right)$ ➜ $\boldsymbol{y = \log_2\left(\frac{1}{3}\right)}$

Written #2

First bullet

Use $y = ab^{t/p}$ to solve for p:

$$0.959 = 1.220\left(\frac{1}{2}\right)^{\frac{10}{p}}$$

Use values from table, after 10 years amount went from 1.220 to 0.959

Isolate power term:

$$\frac{0.959}{1.220} = \left(\frac{1}{2}\right)^{\frac{10}{p}}$$

Then convert to exp. form to solve:

$$\frac{10}{p} = log_{1/2}\left(\frac{0.959}{1.220}\right) \Rightarrow p = \frac{10}{log_{0.5}\left(\frac{0.959}{1.220}\right)} \Rightarrow \boxed{p \approx 28.8 \text{ years}}$$

Half-life

Equation that models amount of Strontium-90:

$$\boxed{y = 1.220\left(\frac{1}{2}\right)^{\frac{t}{28.8}}}$$

Alternatively you could "log both sides"

$$log\frac{0.959}{1.220} = log\left(\frac{1}{2}\right)^{\frac{10}{p}} \Rightarrow log\left(\frac{0.959}{1.220}\right) = \frac{10}{p}log\left(\frac{1}{2}\right) \Rightarrow p = \frac{10log\left(\frac{1}{2}\right)}{log\left(\frac{0.959}{1.220}\right)}$$

Same result, $p \approx 28.8$

Second

Initial percentage is 100, so use $A_0 = 100$

$$10 = 100(0.9172)^t$$

Isolate power term:

$$\frac{10}{100} = (0.9172)^t$$

$$0.1 = (0.9172)^t$$

Then convert to exp. form to solve:

$$t = log_{0.9172}(0.1) \Rightarrow \boxed{t \approx 26.6 \text{ days}}$$

Alternatively you could "log both sides"

$$log0.1 = log0.9172^t$$

$$log0.1 = t\,log0.9172$$

$$\frac{log0.1}{log0.9172} = t$$

Third bullet

"bonus"!

The half-life is the period of time ("p" in the equation) necessary for the initial amount to drop to half.

$$0.5 = 1(0.9172)^t$$

Make the initial amount **1**, *and the end amount* **0.5**

Then convert to exp. form to solve:

$$t = log_{0.9172}(0.5)$$

$$t \approx 8.02 \text{ days} \Rightarrow \boxed{A = 100\left(\frac{1}{2}\right)^{\frac{t}{8.02}}}$$

Alternate equation using half-life of 8.02 days

And with that – you're done!

(Or try another practice exam)